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A new Determination of the Diurnal Rotation of the Planet Mars. By Richard A. Proctor, Esq. B.A.

The determination of a Planet's period of rotation, to any great degree of exactness, may appear a waste of labour. Sir W. Herschel considered such problems worthy of attention, however, as supplying us with a time-measurer *external to the Earth*, and therefore possibly enabling us to detect slow changes in the Earth's rotation. The planet *Mars*, undisturbed by the attractions of a satellite (unless, indeed, some small Moon as yet undiscovered attends upon him), and marked by easily recognisable features, is clearly the one to be selected for such a purpose; and although modern astronomers would hardly look to *Mars* for information respecting such changes, yet, I think, the bare possibility that at some future date the rotation of *Mars* may serve this purpose will make the investigation of the subject interesting to astronomers.

Having by me a large number of pictures of *Mars*, and especially a series of charming tracings kindly given to me by Mr. Dawes, I determined to test the periods assigned by Mädler and Kaiser ($24^{h}\ 37^{m}\ 23^{s}{\cdot}8$ and $24^{h}\ 37^{m}\ 22^{s}{\cdot}6$ respectively). I found that, when the pictures taken by Mr. Dawes in 1852, 1856, 1860, 1862, and 1864, were examined in pairs, the period $24^{h}\ 37^{m}\ 23^{s}$ was perfectly satisfactory. Satisfied, therefore, that this period (or 88643 seconds) was undoubtedly very near the true one, I took the longer intervals. After being misled for a time by fig. 3, plate xvii. in Lardner and Dunkin's *Astronomy*, which seemed to represent the same phase as Dawes' picture of Oct. 3, 1862 (see Lockyer's also of the same date, in vol. xxxii. of our *Memoirs*), I noticed that, with the appended date and hour, that picture (so viewed) was irreconcileable with the picture of Sept. 14th, 1830. Selecting the latter picture as far more marked of the two, I arrived at the conclusion that the period of 88643 seconds was slightly too great. Kaiser's value, 88642·6, seemed, on the other hand, slightly too small. I proceeded to yet longer intervals with the provisional period 88642·8, which I was satisfied was very nearly correct. The remainder of the calculation I will give more at length. So far as I am aware the rotation of *Mars* has never before been tested by such long intervals.

I took, first, Herschel's observation of Sept. 30, 1783, for comparison with Dawes' observation of Oct. 3, 1862. There are several peculiarities about this selection. First, the planet was nearly at the same part of its path (it is noteworthy that 42 revolutions of *Mars* differ by less than a day from 79 revolutions of the Earth); secondly, he presented nearly the same face, and, thirdly, the dates of observation nearly coincide. It will be found that the interval actually separating the two observations (Herschel's made at $10^{h}\ 30^{m}$, Dawes at $12^{h}\ 15^{m}$) is

2493,251100 seconds.

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(It will be remembered that 1800 was not a leap-year.) But there is a correction for phase, since the planet had not quite reached the phase seen by Herschel when Dawes observed it. There is also a slight correction for difference of geocentric longitude, about 4°. Both corrections are additive, and they amount together to about 4000 seconds. The resulting period

2493,255100 seconds,

divided by 88642·8, gives a quotient 28127 *very nearly*. This is the number of revolutions which had elapsed between the two observations. Dividing, therefore, 2493,255100 seconds by 28127, we get the period

88642·76 seconds.

To test this we can apply a much more extended interval. For in the *Phil. Transactions*, vol. i. 1666, there are two pictures taken by Hooke in 1666. They are dated March 3rd, 1665 (0^h 20^m and 0^h 30^m morning), which being translated into our present style becomes March 13th, 1666 (12^m 20^h and 12^m 30^h). These pictures, the only fairly recognisable ones in the set of pictures by Hooke and Cassini (shown in the same plate), represent that long sea running north and south with which Astronomers are familiar.

I found that opposition occurred, in 1666, a day or so after the vernal equinox, and therefore selected, first, a picture taken by Dawes, at 10^h 50^m, April 24th, 1856, in which year opposition occurred on April 2nd. The corrections for phase and geocentric longitude I estimated at − 7385 seconds and + 2462 seconds respectively; and it will be found that the period separating the two observations (taking the earliest of Hooke's) amounts (when thus corrected) to

5999,519276 seconds;

dividing this by 88642·76 seconds we get, *almost exactly*, 67682,—so nearly, indeed, that there is no necessity for the inverse operation.

I took, lastly, Hooke's first observation for comparison with that made by Dawes at 11^h 46^m, Nov. 26th, 1864, an interval of more than 198 years. In this case there was a difference of about 247° in geocentric longitude, and a difference of about 13° in phase, both tending to diminish the interval. These gave 64020 seconds to be subtracted from the interval of 6270,564360 seconds separating the dates of observation. The resulting interval

6270,500340 seconds,

divided by 88642·76 gives 70739 *very nearly*, and this there-

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fore is the number of revolutions of *Mars* which had taken place in the interval. Dividing

$$6270{,}500340 \text{ seconds by } 70739$$

we obtain the rotation period,

$$88642{\cdot}762,$$

which may be assumed to be correct to the second decimal place, since three periods of 79, 190, and 198 years, respectively, agree in giving the same value. Thus we may assign as the true length of the Martial sidereal day,

$$24^{\mathrm{h}}\ 37^{\mathrm{m}}\ 22^{\mathrm{s}}{\cdot}76$$

exceeding our own by

$$41^{\mathrm{m}}\ 18^{\mathrm{s}}{\cdot}67.$$

It is worthy of notice that there cannot possibly (owing to the method of calculation adopted) be an error of one complete revolution of *Mars* in the above period of 198 years. Also Hooke's picture is perfectly recognisable. The only cause of error to be considered is that arising from defective drawing. Assigning an error corresponding to half-an-hour's rotation to Hooke's picture, and an error half as large to Dawes' (he must forgive my assuming so large an error in his beautiful drawing, but I am taking an extreme case), we get a *possible* error of

$$\frac{1800+900}{70739} = \frac{1}{26}\text{th of a second,}$$

very nearly $={\cdot}04$ of a second approximately. But I believe the value obtained differs from the true value by a much smaller amount even than this.

Since the above was written I have gone through the calculations anew, and have detected an error (not in my own work, however) affecting the result by about one-seventieth part of a second.

I had used the synodical period 779·836, given in Nichol's *Cyclopædia of the Physical Sciences*, Mitchell's *Astronomy*, and other works. On testing this value I found it should have been 779·936, and in referring to Loomis' *Practical Astronomy* I find the elements correctly given. The mistake, by a singular accident, corresponds to that which would have arisen if the Earth's tropical period had been used (instead of the sidereal period) in determining *Mars*' mean synodical period. (I say a singular accident, because the mistake is certainly due to misprint only.) Thus the effect of the error is equivalent to the neglect of precession. Instead of *Mars* being in opposition at an epoch determined by the false value, the Earth was in advance of *Mars* by an arc of $50''{\cdot}1 \times t$, where t is the number

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of years between the epoch and that later opposition from which the epoch had been calculated. For instance, instead of *Mars* being in opposition on about the 22nd of March, 1866, a date estimated from the opposition of April 2nd, 1856, the Earth was

$$50''\cdot1 \times 190 = 9919'' = 2\tfrac{3}{4}^\circ \text{ (about)}$$

in advance of *Mars*. This affects the correction for geocentric longitude in the comparison between Hooke's observation and Dawes'. The planet being, on March 13th, 1666, nearly in opposition, and about $\frac{2}{3}$rds as far from the Earth as the Earth from the Sun, we obtain (roughly) a correction of

$$2\tfrac{3}{4}^\circ \times \frac{3}{2} = 4\tfrac{1}{8}^\circ \text{ about}$$

This correction is *subtractive*, and corresponds to rather more than $\frac{1}{90}$th of a Martial day, divided by the number of rotations in about 190 years, that is, to rather more than

$$\left(\frac{88642\cdot3}{90} \div 67650 \text{ seconds}\right)$$

or as nearly as possible to 0·015 seconds, or diminishes the rotation-period to

$$24^{\text{h}}\ 37^{\text{m}}\ 22^{\text{s}}\cdot745,$$

which we cannot consider accurate beyond the second decimal place. I believe the true value lies certainly between

$$24^{\text{h}}\ 37^{\text{m}}\ 22^{\text{s}}\cdot74$$

and

$$24^{\text{h}}\ 37^{\text{m}}\ 22^{\text{s}}\cdot75.$$

It will of course be obvious that, since the whole correction due to the error here detected is only one-seventieth of a second, it would have been useless to have calculated the correction more closely. Also the correction affects all the determinations by about the same amount.

Another effect of the correction is that, instead of 79 sidereal revolutions of the Earth and 42 of *Mars* differing by less than one day, as stated above, the true difference is 2·116 days.

On an Expression for the Angular Distance of two Planets.

By A. Cayley, Esq.

If for the planet m, referred to any fixed plane and origin of longitudes, we have

v, the longitude in orbit
θ, the longitude of node
ϕ, the inclination